Anti-Swing Control of an Overhead Crane by Using Genetic Algorithm Based LQR

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Abstract: Genetic algorithm based optimization of a linear quadratic regulator (LQR) controller which is designed for position and sway control of an overhead crane is presented in this study. Equations of motion of two degrees of freedom (DOF) crane system are derived by using Lagrange formulation and presented as state-space model. A LQR controller is designed by using trial and error method for position control and swing suppression of the crane system. Then, parameters of the LQR controller are optimized by genetic algorithm in order to obtain the best control results. Simulation studies are carried out on a nonlinear crane model which is created in MATLAB/Simulink environment. Performance of the designed controller is evaluated through the simulation results and compared with a pre-designed classical PID controller.

Keywords: Overhead crane, position control, swing suppression, LQR control, genetic algorithm, optimization.

1. Introduction

Cranes have been frequently used for transportation of heavy loads and hazardous materials in industrial fields such as factories, construction sites, harbor etc. Although several types of cranes such as overhead crane, gantry crane, boom crane and tower crane are used for different industrial applications, the problem is same: transporting the load from one point to another as fast as possible without causing any excessive oscillation. But, fast transportation of cable-bound heavy loads causes the excessive oscillations which threatening the operation safety. On the other hand, low operating speeds causes the reduction of operating efficiency. So, position of the cranes must be controlled such a way that the swing of the load is suppressed. For this purpose, numerous studies have been conducted for the position control of cranes as seen in [1]. Since damping capacity of the system plays significant role on the suppression of the swing, passive control methods which the external dampers such as dashpots, viscous dampers, radial spring-damper, etc. are used have been applied to cranes [2,3].

Open loop control schemes such as input shaping [4-6], adaptive input shaping [7] and distributed-mass payload dynamics [8-10] methods have been utilized by researchers. Although the open loop control techniques are cheaper and easy to implement since they do not require the use of sensors, they are very sensitive towards external disturbances since the control input does not account the changes of the system. Several types of closed loop control techniques have been implemented to crane systems for motion and swing control. Proportional Integral Derivative (PID) control has been successfully applied to the control of crane systems as well as in many industrial applications [11,12]. However, there have some difficulties in tuning the PID parameters. Classical tuning methods such as trial and error and Ziegler-Nichols are not guaranteed the significant and satisfactory performances. So, several optimization techniques such as particle swarm optimization (PSO) [13], genetic algorithm (GA) [14] was used for optimization of PID parameters. For achieving a better control performance, the most of the PID controllers were used with other techniques such as neural network-based PID [15], fuzzy PID [16]. Linear Quadratic Regulator (LQR) has been used for the control of crane systems [17]. Fuzzy logic and genetic algorithm was used to optimize LQR parameters [18]. Numerous intelligent controllers have been utilized for crane systems. These controllers have been mainly based on Neural Network (NN) [19-21] and fuzzy controllers [22-24]. Finally, sliding mode control was successfully applied to crane systems in many studies [25,26].

In this study, genetic algorithm (GA) based parameter optimization of a Linear Quadratic Regulator (LQR) which was designed for position control and swing suppression of an overhead crane is presented. Mathematical model of the system is derived by using Lagrange formulation. The equations are linearized by Taylor series expansion and state-space model of the system is obtained. A LQR controller is designed for motion and swing control of the crane system and GA based optimization method is used to determine the best controller parameters. LQR controller designed by using the linearized system model was tested on a nonlinear system model created in MATLAB/Simulink environment. LQR control results are compared with the results obtained from the classical PID control of the same system. Simulation results are presented for analyzing the effectiveness of the LQR controller optimized by GA algorithm.

2. Mathematical Modelling of Overhead Crane

In this study, overhead type crane system is selected for control. An overhead crane operates on elevated runway beams along the production line. Generally, it consists of three main parts: girder, trolley and cable. Trolley is mounted to girder in orthogonal position. While the girder moves backward and forward, it is possible to move the trolley right and left side. Besides, the cable can move in vertical direction for lifting up and down the load. The simplified model of the overhead crane system considered in this study is given in Fig. 1. Motion of the girder is not considered since there is no coupling between
the motions of girder and the trolley. In addition, the length of the cable is assumed to be constant. In Fig. 1, $\theta(t)$ is the sway angle of the load, $x(t)$ is the displacement of the trolley, $L$ is the length of the cable, $m_T$ is the mass of trolley, $m_P$ is the mass of payload and $F(t)$ is the control force applied to the trolley. The system parameters are given in Table 1.

**Figure 1.** Simplified model of overhead crane

**Table 1.** System parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_T$ = 3.9 kg</td>
<td>Mass of the trolley</td>
</tr>
<tr>
<td>$M_P$ = 1 kg</td>
<td>Mass of the payload</td>
</tr>
<tr>
<td>$L$ = 0.3 m</td>
<td>Length of the cable</td>
</tr>
<tr>
<td>$g$ = 9.81 m/s$^2$</td>
<td>Acceleration of gravity</td>
</tr>
</tbody>
</table>

Lagrange formulation is used to obtain mathematical equations of the system. Lagrangian of the system can be defined as,

$$ L = T - V $$

(1)

where $T$ and $V$ are the sum of the kinetrical and potential energies of the cart and pendulum respectively. Lagrange formulation is,

$$ \frac{d}{dt} \left( \frac{\partial L}{\partial q} \right) - \frac{\partial L}{\partial q} = Q_i $$

(2)

where $q_i$ is the generalized coordinates and $Q_i$ is the generalized forces of the system. For this problem, the generalized force is $F$ and the generalized coordinates are,

$$ q(t)^T = [x(t) \; \theta(t)] $$

(3)

where $x(t)$ is the position of the cart in the horizontal direction and $\theta(t)$ is the angle of the pendulum relative to the vertical direction.

If the kinetic and potential energies of the system are written in Eq. 1, Lagrangian of the system can be obtained as below.

$$ L = \frac{1}{2} (m_T + m_P)\dot{x}^2 + \frac{1}{2} m_PL^2 + m_P \dot{x}L \cos \theta - m_PgL $$

$$ + m_PgL \cos \theta $$

(4)

If the Lagrangian of the system is written in Eq. 2, the equations of motion are obtained as follows.

$$ (m_T + m_P)\ddot{x} + m_PL \dot{\theta} \dot{x} - m_PL \sin \theta \dot{\theta}^2 = F $$

(5)

$$ m_PL \cos \theta \ddot{x} + m_PL^2 \ddot{\theta} + m_P L \dot{\theta} + m_P gL(1 + \sin \theta) = 0 $$

(6)

If $x$ and $y$ denotes the states and outputs respectively, state-space representation is defined as follows.

$$ \dot{x} = Ax + Bu $$

$$ y = Cx + Du $$

(7)

States ($x$) and outputs ($y$) of the system can be described as follow.

$$ x^T = [x_1 \; x_2 \; x_3 \; x_4] = [x \; \theta \; \dot{x} \; \dot{\theta}] $$

$$ y^T = [x_1 \; x_2] $$

(8)

If the equations of motion are linearized around the equilibrium point by using Taylor-series expansion with the assumption of small oscillations and written in Eq. 7-8, state and output matrices of the system are obtained as below for the system parameters given in Table 1.

$$ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -13.907 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -42.783 & 0 \end{bmatrix} $$

$$ B = \begin{bmatrix} 0 \\ 0.2142 \\ 0 \\ 0.3954 \end{bmatrix} $$

(9)

$$ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} $$

(10)

3. Genetic Algorithm Based Optimization of LQR Controller

The overhead crane system is a good example of an underactuated system which two different system variables are tried to be controlled by a single input. However, this property makes the control problem harder due to complex dynamics. The aim of the controller is minimizing the positioning error as fast as possible while suppressing the load swing. Linear-Quadratic-Regulator (LQR) is a control technique widely used in many control applications.

In the design of LQR controller, performance index $J$ is used for calculating the optimal control inputs. The $J$ performance index including the state errors and the system inputs is given in Eq. 11. System inputs which minimize the performance index $J$ are calculated by using $Q$ and $R$ diagonal matrices given in Eq. 12.

In the LQR control, system input is expressed as $u = K_t(\text{ref} - x)$ for $\dot{x} = Ax + Bu$. LQR gain is expressed as $K = R^{-1}B^TP$. In this equation, $P$ is a symmetrical matrix obtained from Ricatti Equation given in Eq. 13.

$$ J = \int_0^\infty (q_{ref} - q(t))^T Q (q_{ref} - q(t)) + u(t)^T R u(t) \; dt $$

(11)
Trial and error method is generally used in determination of LQR parameters. In addition to being extremely laborious, it is almost impossible to find the best parameters with this method. So, many optimization methods such as genetic algorithm, the bees algorithm, particle swarm optimization, etc. are used for determining the optimum LQR parameters. Genetic Algorithm (GA) is an optimization method proposed by John Holland in 1970 for improving the performance of computational methods. Besides being simple and robust, GA requires only the fitness function and does not need any information about the system. It searches the best solution by using a number of individuals in parallel instead of a single solution. Flowchart of the GA is given in Fig. 2.

In the starting, initial population consisted of random chromosomes is created. Each chromosome provides a solution to problem. Fitness value of each chromosome is calculated by using fitness function and a number of chromosomes which given the best fitness values are selected. After the selection, each chromosome undergoes mutation and then reproduce or crossover. The chromosomes that undergo mutation give better results than previous and so being closer to best solution after each mutation.

LQR control scheme of the system is given in Fig 3. Inputs of the system are the desired positions of the trolley ($x_{\text{ref}}$) and the payload ($\theta_{\text{ref}}$) and outputs are the actual positions and velocities of the trolley ($x$, $\dot{x}$) and payload ($\theta$, $\dot{\theta}$).

Parameters of $Q$ and $R$ matrices of pre-designed LQR controller are optimized by using GA. The values for minimizing the objective function of the system have been investigated. The objective function used for the optimization of the LQR controller is given in Eq. 14. The objective function includes the parameters of peak time ($t_p$), rise time ($t_r$), settling time ($t_s$), steady state error ($x_{\text{sse}}$) and maximum peak ($x_{\text{max}}$) which are obtained from the step response of the system. Selected intervals for the optimization are given in Table 2.

$$J_{e} = (x_{\text{tr}} + x_{\text{ts}} + x_{\text{tp}} + x_{\text{max}} + x_{\text{sse}}) + (\theta_{\text{norm}} + \theta_{\text{ts}})$$

$$+ \theta_{\text{tp}} + \theta_{\text{max}} + \theta_{\text{sse}}$$

Table 2. Intervals for optimization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Max</td>
<td>150</td>
<td>10</td>
<td>150</td>
<td>10</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

4. Simulations and Results

Optimization of the LQR controller designed for the position and sway control of an overhead crane was performed by using GA. The parameter values determined for the optimization are given in Table 3. The weight matrices ($Q$ and $R$) and the gain matrix ($K$) calculated for the LQR controller are given in Eqs. 15, 16 and 17 respectively.

Table 3. Optimization Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>GN = 60</td>
<td>Generation number</td>
</tr>
<tr>
<td>PS = 30</td>
<td>Population size</td>
</tr>
<tr>
<td>EC = 15</td>
<td>Elite count</td>
</tr>
<tr>
<td>CF = 0.6</td>
<td>Crossover fraction</td>
</tr>
</tbody>
</table>

$$Q = \begin{bmatrix} 148.6478 & 0 & 0 & 0 \\ 0 & 0.0405 & 0 & 0 \\ 0 & 0 & 36.8234 & 0 \\ 0 & 0 & 0 & 6.6425 \end{bmatrix}$$

$$R = 0.0308$$

$$K = [69.4976 \ 44.5258 \ 32.9022 \ -0.1793]$$
Simulation studies are carried out on the nonlinear overhead crane model created in the Matlab/Simulink environment. Performance of the optimized LQR controller is compared with a classical PID control. Time-dependent changes of the trolley position and sway angle in cases of 0.5 m, 1 m and 1.5 m step inputs are given in Fig. 4-6 respectively. Simulations show that the optimized LQR controller gives better results than the PID controller both in terms of settling time and the swing amplitude. The LQR controller successfully eliminates the oscillations occurred in the trolley motion in case of PID control. Furthermore, the LQR controller reduces the swing amplitude occurred in case of PID control by about half. Finally, the LQR controller eliminates the steady state error occurred for 1.5 m step input in case of PID control.

5. Conclusions

GA based optimization of a LQR controller designed for position and sway control of an overhead crane system is presented in this study. A LQR controller was designed by using state-space model of the system. The parameters of LQR controller (Q and R matrices) are optimized by using GA. An objective function including the parameters of peak time ($t_p$), rise time ($t_r$), settling time ($t_s$), steady state error ($x_{ss}$) and maximum peak ($x_{max}$) which are obtained from the time...
response of the system was used the optimization process. The performance of the optimized LQR controller has been investigated through the simulation studies realized in MATLAB/Simulink environment and compared with the results obtained from classical PID control. Simulation results showed that the optimized LQR controller presents better control results than the PID controller both in terms of settling time and the swing amplitude.

References


