Performance Evaluation Analysis Of An Interference Alignment Based Precoding In Mimo-Ofdm System
Chanda Sivakantha Reddy (PG Scholar)\textsuperscript{1} B. Naga Rajesh Assistant Professor\textsuperscript{2} K. Chandra Sekhar Assistant Professor\textsuperscript{3}

Department of ECE, Chaitanya Bharathi Institute of Technology, Proddutur, AP, India

Abstract

Although tremendous progress has made in the past years to eliminate the inter-block interference (IBI) but still it is considered as an area of concern in orthogonal frequency division multiplexing system. We proposed interference alignment based channel independent precoding in MIMO OFDM system. We showed that when the number of receive antennas ($n_r$) is not more than the number of transmit antennas ($n_t$), our proposed precoding is more bandwidth efficient than the conventional zero-padded or CP added MIMO systems, such as, ZP-only, CP-OFDM and SC-FDE system and When the number of receive antennas is more than the number of transmit antennas, it was shown that the IBI in an MIMO OFDM system can be completely eliminated without any CP or zero-padding or precoding, when the OFDM block size is not too small. In this paper, we consider only CP based block transmission system.

KEYWORDS: MIMO-OFDM, Precoding, Cyclic prefix, IBI

1. INTRODUCTION

OFDM system is Orthogonal Frequency Division Multiplexing system where the sub-carriers are orthogonal to each other. This indicates, non-overlapping of sub-carriers, which results in efficient spectrum utilization. Along with efficient spectrum utilization OFDM has high data rate as one of its advantages. But, due to emphasizing of sub-carriers in same direction high peaks are achieved resulting in high PAPR. This high value of PAPR destroys the orthogonality of the sub-carriers resulting in destruction of the OFDM system. Therefore, it is essential to reduce the PAPR value of the OFDM system. Various efforts are made to reduce PAPR value using different techniques. PAPR reduction techniques are basically classified as distorted and un-distorted techniques. The distorted techniques results in distortion but have no data rate loss while un-distorted techniques have distortion less data but results in data rate loss.

The combination of MIMO signal processing with OFDM is considered as one of the most promising techniques for enhancing the data rate of next-generation wireless communication systems. OFDM divides a broadband signal into multiple narrowband subcarriers, where each subcarrier is more robust to multipath. In order to maintain orthogonality among subcarriers, a CP is added at the head of each symbol. In MIMO-OFDM system, insertion of IDFT and CP at the transmitter and removal of CP and DFT at the receiver together it help to convert an inter-symbol interference channel into several ISI free sub channels. The CP length is designed which is not less than the length of the channel impulse response (CIR) in order to eliminate the effects of the inter block interference (IBI) and inter-carrier interference (ICI). A considerably long CP is needed if the multipath delay spread is large, resulting in a substantial loss in both bandwidth and power efficiencies.

In a MIMO-OFDM system with insufficient CP, if the IBI from the previous OFDM block can be separated and eliminated, it will be easier to detect the current OFDM block from the desired signal term and the ICI term both of which contain the information of the current OFDM symbol.
Interference alignment (IA) provides a novel concept to deal with interferences. The basic idea of IA is to use well-designed “beam forming” vectors at the transmitter such that the interference vectors are aligned at the receiver in one subspace which is disjoint from the signal subspace. As a result, the interference vectors are separated from the desired signal subspace and are limited in the minimum dimensions and therefore can be eliminated by the zero-forcing operator at the receiver. This basically provides an interference nulling technique.

The quality of a wireless link can be described by three basic parameters, namely the transmission rate, the transmission range and the transmission reliability. Conventionally, the transmission rate may be increased by reducing the transmission range and reliability. By contrast, the transmission range may be extended at the cost of a lower transmission rate and reliability, while the transmission reliability may be improved by reducing the transmission rate and range. However, with the advent of MIMO assisted OFDM systems, the above-mentioned three parameters may be simultaneously improved.

2. SYSTEM MODEL

2.1 BASIC OFDM SYSTEM

The block diagram of OFDM system is shown in fig.1. The input high data rate streams are converted into number of low data rate streams. This parallel stream is then modulated using QPSK or QAM modulation techniques, which is then applied as input to IFFT block producing OFDM samples. These samples are then converted into OFDM signal using Parallel-to-Serial converter (P/S). The signal is then encoded by adding Cyclic Prefix (CP) and is then transmitted over the channel. The reverse process is done at the receiver

2.2 MIMO OFDM SYSTEM

The following block diagram is of MIMO-OFDM system which consists multiple transmit antenna and multiple receive antenna. In this diagram, the binary source generates digital input data sequence. This binary data is encoded by using digital modulation scheme like BPSK, QPSK and QAM with several different constellations. The serial to parallel block performs data symbols parallelized in N different sub streams. Each sub stream will modulate a separate carrier through the IFFT modulation block. The IFFT block converts parallel sub-streams of frequency domain data symbols into a time domain OFDM symbol. After that we have to insert cyclic prefix to remove inter-block interference and inter-symbol interference. The cyclic prefix means we have to copied last specific length of data bits in start of OFDM symbol. The data are back convert into parallel to serial form. This serially converted data is transmitting from multiple transmit antenna to multiple receive antenna through the channel and AWGN noise is added in received OFDM symbol through the channel.

![Figure 1: Block diagram of Basic OFDM system](image-url)
The received data are first converted in serial to parallel form and also remove cyclic prefix. The FFT block converts this data from time domain symbol in frequency domain and again covert it parallel to serial form. When we have to demodulate and decode the data, we get estimated output. The MIMO-OFDM system has multiple input and multiple output.

### 2.3 SISO OFDM SYSTEM

In SISO-OFDM system we consider N subcarriers with frequency selective fading channel and which is represented by a vector \( h = [h(0), h(1), \ldots, h(L)]^T \), where L is order of CIR and \( L + 1 \) is length of channel impulse response and we assume \( N \geq L \) in this paper. The input signal vector of kth OFDM block is \( r_k = [r_k^0, r_k^1, \ldots, r_k^{N-1}]^T \). Let \( W_N \) is the normalized IDFT matrix of size N with entries. The IDFT operation is performed at the transmitter and it converts the input signal from frequency domain to time domain. We add a cyclic prefix \( v \) to each time domain vector which is insufficient, \( V \leq L \). The transmitted OFDM block is affected by both ICI and IBI components. After the insufficient CP is removed at the receiver, the time domain expression of the kth received OFDM block is given.

\[
y_k = (H - A)W_N^*r_k + BW_Nr_{k-1} + n_k \quad (1)
\]

\( A = \begin{bmatrix} 0_{(L-v) \times (N-l)} & S & 0_{(L-v) \times v} \\ 0_{(N-l+v) \times (N-L)} & 0_{(N-L+v) \times v} \\ 0_{(N-L+v) \times (N-L+v)} & 0_{(N-L+v) \times v} \end{bmatrix} \quad (2)
\]

\( B = \begin{bmatrix} 0_{(L-v) \times (N-L-v)} & S \\ 0_{(N-L-v) \times (N-L+v)} & 0_{(N-L-v) \times v} \end{bmatrix} \quad (3)
\]

\[
S = \begin{bmatrix} h(L) & h(L-1) & \cdots & h(v+1) \\ 0 & h(L) & \cdots & h(v+2) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \quad (4)
\]

In the above equations (2) and (3), matrices A and B are the time domain expressions derived under the assumption of perfect synchronization and a rectangular pulse shape. If CP length \( v \) is larger than or equal to the CIR order \( L \), A and B is both the all zero matrices means no ICI or IBI exists in the received signal.

At the receiver, the time domain signal \( y_k \) in (1) is converted into the frequency domain signal \( z_k \)

\[
Z_k = W_N^{-1}CW_Nr_k + W_N^{-1}BW_Nr_{k-1} + \tilde{n}_k \quad (5)
\]

Now we have to see precoding, so signal passing through a precoding matrix \( P \) of size

\[
r_k = Px_k \quad (6)
\]

The received frequency domain signal for the kth OFDM block can be equivalently expressed as

\[
Z_k = W_N^{-1}CW_kPx_k + W_N^{-1}BW_kPx_{k-1} + \tilde{n}_k \quad (7)
\]

### 2.3 MIMO OFDM SYSTEM
The input to the MIMO-OFDM system is denoted by \( \bar{r}_k = [(r_k^0)^T, (r_k^1)^T, \ldots, (r_k^{N-1})^T]^T \) where \( r_k^l \) denotes the \( n_t \) transmit antennas for the \( i_t \) vector for the \( n_t \) transmit antennas at the \( n_t \) subcarrier, \( 0 \leq l \leq N - 1 \), in frequency domain. Next, the input vector \( \bar{r}_k \) is transformed into time domain signal by \( n_t \) IDFT matrices of size \( N \times n_t \) transmit antennas. The overall IDFT operation over \( \bar{r}_k \) can be represented by \( \bar{r}_k \) \( \bar{W} = W_N \otimes I_{n_t} \). At each transmit antenna, a CP of length \( v \) is added to the input signal block and propagates via multipath channel \( h_{ij} = [h_{ij}(0), h_{ij}(1), \ldots, h_{ij}(L)]^T \) in between the \( i \)th receive antenna and the \( j \)th transmit antenna, where we assume that all the entries of \( h_{ij} \) are i.i.d. complex Gaussian random variables with 0 mean and the channel length, \( L + 1 \), is identical for all the channels. We now define \( n_r \times n_t \) channel matrices \( H(l) \), \( l = 0, 1, \ldots, L \), as

\[
H(l) = \begin{bmatrix}
h_{11}(l) & \cdots & h_{1n_r}(l) \\
\vdots & \ddots & \vdots \\
h_{n_r1}(l) & \cdots & h_{n_rn_r}(l)
\end{bmatrix}
\]

These matrices \( H(l) \), \( l = 0, 1, \ldots, L \), are the multipath channel matrices for the time domain vectors \( r_k^l \) serially transmitted at \( n_t \) transmit antennas. Due to the randomness of the channel coefficients, all the matrices \( H(l) \) are of full rank almost surely. At the receiver, the CP is removed and the overall time domain received block is given

\[
\bar{y}_k = \bar{C} \bar{W} \bar{r}_k + B \bar{W} \bar{r}_{k-1} + n_k
\]

Where \( n_k \) is the \( N \times 1 \) noise vector with the complex Gaussian distribution \( \mathcal{CN}(0, \sigma^2 \mathbf{I}) \), \( C \) and \( B \) of size \( N \times n_r \) and \( N \times n_t \) are the overall channel matrix and IBI matrix, respectively, constructed by stacking sub matrices \( H(l) \)

\[
B = \begin{bmatrix}
0 & 0 & \cdots & H(L) & \cdots & H(v + 1) \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \cdots & \cdots & H(l) \\
0 & 0 & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \cdots & \cdots & 0 \\
\end{bmatrix}
\]

Before the signal detection, the DFT operation \( W_N^{-1} \otimes I_{n_r} \) is applied to \( \bar{y}_k \) yielding the received signal \( \bar{z}_k \) in frequency domain. For this MIMO-OFDM system, the input vector \( \bar{r}_k \) is the time domain block of output of information symbol vector by an \( n_t \times N \times n_t \) precoding matrix \( P \), where \( x_k^l \) is the \( n_t \times 1 \) information symbol vector associated with \( r_k^l \), \( \bar{r}_k = P \bar{x}_k \)

\[
\bar{r}_k = \begin{bmatrix}
W_N^{-1} \otimes I_{n_r} \end{bmatrix} C Q \bar{x}_k + \begin{bmatrix}
W_N^{-1} \otimes I_{n_r} \end{bmatrix} B Q \bar{x}_{k-1} + \bar{n}_k
\]

\[
3. \ \text{CHANNEL INDEPENDENT PRECODING}
\]

A precoding is designed to eliminate the distortion by processing information symbols at the transmitter and it also requires the perfect CSIT.

3.1. SISO-OFDM precoding

The equation (1) we have to change like equation (7) using precoding so we obtain following equation

\[
y_k = C Q x_k + B Q x_{k-1} + n_k
\]

For the current \( k \)th OFDM block, the signal \( x_k \) and \( B Q x_{k-1} \) is the IBI and we assume the additive noise \( n_k \) is negligible. In order to freely solve for \( x_k \) from (14), the space \( \mathbf{v}_{\text{signal}} \) linearly spanned by the column vectors of \( C \) and the space \( \mathbf{v}_{\text{IBI}} \) linearly spanned by the column vectors of \( B Q \) need to be disjoint. The determinant of channel matrix \( C \) is zero and its rank is full i.e. \( N \). we assume rank of precoder \( Q \) is \( N - d \) so the rank of IBI matrix \( B Q \) becomes \( L - v - d \) and rank of \( C \) is \( N - d \). The sum of rank of \( IBI \) and \( C Q \) is not more than the vector size \( N \)

\[
N - d + L - v - d \leq N
\]

\[
d = \frac{l - v}{2}
\]

The above equation shows dimensions of space is spanned by the IBI is \((L - v)/2\). In conventional In the conventional OFDM system (or unprecoded OFDM system), additional \( L - v \) zeros or redundant symbols are needed to make the IBI disappear. From the above analysis, only half of \( L - v \) zeros or redundant symbols are needed to separate the spaces of the signal and the IBI for the signal to be solved freely \[1 \]. We see now a particular example for the above SISO-OFDM precoding idea, let us consider the case when \( N = 64 \) subcarriers, CIR length \( L + 1 = 17 \), i.e., \( L = 16 \), and the insufficient CP length \( v = 8 \). So rank (CQ) = 60, rank(BQ) = 4. In this example, 60 independent information symbols can be solved freely. With CP length \( v = 8 \), in the conventional OFDM of block size \( N = 64 \), 8 more zeros or redundant symbols in the OFDM block are needed to completely
eliminate the IBI, and thus only 60 independent information symbols are included.

3.2. MIMO-OFDM Precoding

In MIMO-OFDM system, we have to see the IBI term from previous OFDM block needs to be prevented and current OFDM block should be preserved. The MIMO-OFDM precoding is same as like SISO-OFDM precoding which is explained subsection. In next subsection, we have to consider problem in two different cases for the numbers of transmit and receive antennas

3.2.1 Precoding Technique when \( n_r \leq n_t \) (Theorem 1)

In MIMO-OFDM system, \( n_t \) transmit antennas; \( n_r \) receive antennas \( N \) subcarrier so for each OFDM block there are total \( n_tN \) linear equations after the removal of CP. In order to linearly solved for all information symbols, the number independent information symbols passed through \( n_t \) transmit antennas should be not more than \( n_tN \). The following theorem is solved for number independent information symbols using zero-forcing operator during each OFDM block.

\[
\begin{align*}
    n_t(N - L + v) + \left\lfloor \frac{n_rN - n_t(N - L + v)}{2} \right\rfloor, \\
    \text{if } n_t(N - L + v) < n_rN, \\
    n_rN, \\
    \text{if } n_t(N - L + v) \geq n_rN.
\end{align*}
\]

The above equation is for insufficient CP MIMO-OFDM system with \( n_r \leq n_t \).

3.2.2 IBI cancellation when \( n_r > n_t \) (Theorem 2)

Using following theorem we show that to eliminate IBI there is no precoding or no CP or no zero padding are needed. The total number of independent information symbols can be solved using the zero-forcing operator is \( n_rN \) for the insufficient CP MIMO-OFDM system with \( n_r > n_t \), where no zero-padding or precoding is needed.

\[
N \geq \frac{n_r}{n_r - n_t}(L - v) \quad (18)
\]

4. SIMULATION RESULTS

![Figure 3: BER performance of IA based precoding lengths and different numbers of information’s (Symbols=32)](image)

![Figure 4: BER performance of IA based precoding lengths and different numbers of information’s (N=32, L=12, nt=4, nr=2 Symbols=128)](image)

![Figure 5: BER performance of IA based precoding lengths and different numbers of information’s (N=64, L=16, nt=1, nr=1 Symbols=64)](image)

![Figure 6: BER performance of IA based precoding lengths and different numbers of information’s (N=32 L=19, nt=4, nr=2 Symbols=64)](image)
Figure 7: BER performance of IA based precoding lengths and different numbers of information’s (N=64 L=16, nt=1, nr=1 Symbols=64)

5. CONCLUSION

In this paper, we proposed that the interference alignment based channel independent precoding in MIMO-OFDM system with insufficient CP and we conclude also when number of receive antennas are more than number of transmit antennas, we can eliminate total IBI using precoding technique (using Theorem 1). We also explained the total SISOOFDM model and MIMO-OFDM model and their precodings in detail. Our proposed precoding’s bandwidth efficiency is more than the conventional zero-padded or CP added MIMO systems, such as, ZP-only, CP-OFDM and SC-FDE system when nr ≤ nt.

6. REFERENCES


[16]. Jan Tubbax†, Boris C’ome, Liesbet Van der Perre, Luc
Deneire, St´ephane Donnay, Marc Engels_IMEC -
Kapeldreef 75, 3001 Heverlee, Belgium “Compensation of
IQ imbalance in OFDM systems”.0-7803-7802-4/03/$17.00
© 2003 IEEE.

[17]. Marcus Windisch, Gerhard Fettweis Dresden University
of Technology, Vodafone Chair Mobile Communications
Systems, D-01062 Dresden, Germany “On the Impact of I/Q
Imbalance in Multi-Carrier Systems for Different Channel
Scenarios”.

Chanda Sivakantha Reddy
has completed Bachelor of Engineering-(ECE)
from ANNA UNIVERSITY, CHENNAI in
the year 2011 and currently pursuing the Master of
Technology in the field of Electronics and
Communication Engineering from JNTU
Anantapur. His area of interests include
Communication, Mobile communication, Signal
processing.

B.Naga Rajesh has completed B.E-(ECE) from Jawaharlal Nehru
Technological University (HYDERABAD) in
the year 2008 and completed the M.tech from
JNTU (Anantapur) in the year 2011. He has five
years of academic experience as assistant
professor and currently registered for the Ph.D
from the KL UNIVERSITY. His area of
interests Digital Image/Signal Processing and
Wireless Communication and Networks

B.Naga Rajesh has completed B.E-(ECE) from Jawaharlal Nehru
Technological University (HYDERABAD) in
the year 2006 and completed the M.tech from
JNTU (Anantapur) in the year 2012. He has
three years of academic experience as assistant
professor and currently he is working as assistant
professor in CBIT proddatur. His area of
interests includes VLSI Design and Digital
Electronics