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Analysis of Natural Frequencies and Vibrations in Pipes Under Fluid Flow Using Theoretical Modeling

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Abstract

Piping systems are effective applications in various industrial sectors due to their widespread use in fluid transportation. Given their key role in infrastructure and industrial processes, it has become imperative to focus on the study of piping and the technical problems they may encounter during operation. These important and complex problems Which cause failures in many pipe networks are the problem of vibrations. Where researchers have dealt with the past years the subject in many respects, but the problem has not ended so far. Therefore, in this research, the problem of controlling the vibration resulting from the flow of fluids inside the pipes was shed light on, in addition to the study of vibration reduction for these pipes, where the research included studying the natural frequency of pipes of various types. Fixation to the pipe at a constant speed and different pressure for two types of fluids (water and oil) at a certain length and diameter, and observing the effect of this addition on the amount of vibration in addition to the natural frequency of the pipe. This study was done by deriving differential equations for pipes for different types of fixations (Pinned-Pinned, Clamped-Pinned, Clamped-Clamped) and the application of boundary conditions for each equation.

Where the equations for the natural frequency of the tubes were reached by the theoretical method (analytically) by deriving the differential equations of motion and dealing with them for the purpose of arriving at the general equation so that it is easy to convert it with changing the inputs and outputs and using the Matlab 2018 program, where the final equations were graphed and the results for the tubes were measured.

Keywords: Natural Frequency; Velocities; Pressures; vibration; Pipe conveying fluid.

Nomenclature	Meaning
D_o	Outside Diameter
D_i	Inner Diameter
E	Modulus of elasticity
k_0, k_1, k_2, k_3, $k_4 - k_5$	constants
L	length
m_f	Mass of fluid
m_p	Mass of pipe
$q_i(\tau)$	generalized coordinate
Re	Reynolds number
$\lambda_1, \lambda_2, \lambda_3$	pipe eigenvalues
Π	pressure
u_o	Speed
Ω	natural frequency of pipe (unitless)
ω	natural frequency of pipe

$\boldsymbol{\rho}$	**density**
$\boldsymbol{I}$	**Second moment of area**
$\boldsymbol{m}$	**Mass**
$\boldsymbol{\eta}$	**Loss factor**
$\boldsymbol{\rho}$	**Density of pipe**
$\boldsymbol{\mu}$	**dynamic viscosity of the fluid**

1. Introduction

Piping systems used for fluid transport are a vital component of many advanced engineering applications, including aerospace, oil transportation, deep-sea exploration, nuclear power projects, and other diverse applications. However, the efficiency of these systems is significantly limited by the complex dynamic interaction between the fluid and the tubular structure. This interaction is driven by a combination of factors, such as the complexity of structural and functional designs, supporting and support conditions, internal flow characteristics, and influences from the surrounding external environment. [1]. Fluid flow is divided into two main types: internal flow and external flow, depending on whether the fluid is moving within a closed channel or over an external surface. Each of these two types has different dynamic characteristics. In internal flow, the fluid moves within a closed, fully filled pipe or channel, and the flow is driven by a pressure difference within the channel. In external flow (open channels), such as irrigation canals and the air around an airplane wing, the fluid does not completely fill the channel, resulting in contact with solid surfaces and a free surface. In this case, the flow is primarily driven by gravitational forces rather than pressure differences.[2].

The main objective of the research is to find control over the vibration of the tubes resulting from the flow of fluid inside the tube. Where the control process is done by increasing the natural frequency of the tube by changing several factors, including the change in pressure at a constant speed of the fluid inside the tube and the change in the type of fluid. This study is to solve the problem in several ways, including analytical by solving the differential equations of the subject and the change of several factors affect the vibration as shown, The main axes included in the research:

1- Find a study of previous research, which dealt with topics close to the subject of research, it is done by finding research and studies that deal with topics close to the subject of our research and reading and know the results reached until the conclusions and accurate information for the research.
2- To find a theoretical analysis through the derivation of the differential equations of the tubes, this is done by rounding up the tubes and using appropriate equations, selecting suitable cases of stabilizers, by applying the appropriate boundary conditions for each of the cases addressed to determine the natural frequency.

The following are the types of installation used:

a. Pinned –Pinned
b. Clamped-Clamped
c. Clamped-Pinned

3- Calculating the amounts of natural frequency at constant speed and variable pressure (2bar ,4bar ,6bar) for each type of pipe fixing for different fluid (water and oil) and comparing the theoretical results.
4- Matlab 2018 program is used to program equations for each type installation at a constant speed and variable pressure for each case and to find the results accurately. Two types of fluids are used, water and oil. As for the length of the tube, it is fixed in all cases and equal to 1.5m.

Vibration of fluid transport pipes is a profound challenge in many fields and throughout the ages, So the hot topic started when studied shley and Haviland [3,4] studied the flow-induced vibration of the transArabian pipeline. This introductory study was followed by a series of other studies through which the nature of the vibration was deeply investigated and then analyzed theoretically and analytically. Paidoussis [5] demonstrated that fluid flow within pipes can induce vibrations that result in two different types of dynamic instabilities. The first type is “divergence instability,” which typically occurs at relatively low flow velocities, while the second type, “flutter instability,” appears at higher velocities and is characterized by more complex vibrational behavior. Benjamin [6] discussed the phenomenon of flutter instability in pipes clamped at one end through which a fluid is transported. Paedousis and Lee [7] made a detailed study of the instability in cylindrical pipes resulting from the influence of a fluid, using linear vibration equations. In their research, they addressed two types of instability problems: buckling and flutter. Paedousis and Essid [3, 8] proposed a number of methods for analyzing the dynamic behavior of fluid-containing pipes. Several techniques have been developed to address the problem of vibration of these pipes in linear and nonlinear models, the most prominent of which are the Galerkin method and the finite element method (FEM). Y. M. Huang [9] studied the natural frequencies generated by the structure-fluid interaction in fluid-conducting pipes, using the Galerkin method with neglected elements as an analytical tool. Through this study, natural frequency equations were derived under different boundary conditions, contributing to a deeper understanding of the dynamic behavior of structural systems affected by fluid flow. Hausner [3,10] analyzed the vibrations caused by fluid flow in pipes with pinned-pinned boundary conditions, using analytical methods to determine their dynamic behavior. Ibrahim [11, 12] and Li et al. [13] provided a comprehensive and systematic summary of research on fluid-conducting pipes. Dai et al. [14] also studied vibrations in a flexible pipe conveying oscillating currents, where the main factors influencing vibrational behavior were analyzed and discussed. In this context, the focus of this study is on the issue of vibration resulting from steady flow inside a pipe conveying a fluid with a constant internal velocity.

2. Methodology

In this section, This study provides a theoretical analysis of the dynamic and vibrational behavior of fluid-transporting pipes. To clarify the concept, the dynamics and stability of fluid-transporting pipes will be reviewed, followed by the active vibration control of these systems. The study aims to provide a comprehensive understanding of the dynamic behavior of pipes under fluid flow by applying several analytical methodologies. Based on previous studies, this study will focus on analyzing the equilibrium (zero-equilibrium) stability of a pipe under pulsating flow.

2.1. Linearized Equations of Motion and Evaluation of the Natural Frequencies

In this section, the vibration characteristics of fluid-conveying pipes will be analyzed for both conservative and non-conservative cases. The primary objective of this analysis is to estimate the natural frequencies of these pipes. To achieve this, it is necessary to derive the linear equations of motion in the vicinity of the equilibrium position. By neglecting the effects of pulsating flow and external forces, and assuming the system is in a steady state, the equation of motion can be simplified into the following linear form:

$$\ddot{\eta} + 2M_r u_o \dot{\eta}' + \left[u_0^2 + \Pi\right]\ddot{\eta} + \eta^{(4)} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The equation (1) is inhomogeneous where the derivative coefficients of η are frank functions of τ and ξ then the discretized equation of motion above, by using the Galerkin's way let.

$$\eta(\xi,\tau) = \sum_{i=1}^{\infty} \emptyset_i(\xi) q_i(\tau) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$q_i(\tau)$ is a generalized coordinate, $\emptyset_i(\xi)$ is a comparison function which satisfies all the boundary conditions? Selecting the first three orders conducts researches, which is:

$$\eta(\xi,\tau) = \sum_{i=1}^{3} \emptyset_i(\xi) q_i(\tau) = \emptyset_1(\xi) q_1(\tau) + \emptyset_2(\xi) q_2(\tau) + \emptyset_3(\xi) q_3(\tau) \ldots\ldots\ldots (3)$$

For pipes pinned at both ends, the function of its vibration model is:

$$\emptyset_i = \sqrt{2}\, sin(\lambda_i \xi), \qquad i = 1,2,3$$

$\lambda_1 = \pi$, $\lambda_2 = 2\pi$, $\lambda_3 = 3\pi$ **, where** λ_1, λ_2 *and* λ_3 are pipe eigenvalues.

For pipes fixed at both ends, the function of its vibration model is:

$$\emptyset_i = cosh(\lambda_i \xi) - cos(\lambda_i \xi) + \frac{cosh(\lambda_i) - cos(\lambda_i)}{sinh(\lambda_i) - sin(\lambda_i)}\left[sin(\lambda_i \xi) - sinh(\lambda_i \xi)\right] \quad , i = 1,2,3 \ldots (4)$$

$$where\ \lambda_1 = \frac{3\pi}{2} \quad , \quad \lambda_2 = \frac{5\pi}{2} \quad , \quad \lambda_3 = \frac{7\pi}{2}$$

For pipes pinned at one end and fixed at another end, the function of its vibration model is:

$$\emptyset_i = cos(\lambda_i \xi) - cosh(\lambda_i \xi) + \frac{cos(\lambda_i) - cosh(\lambda_i)}{sin(\lambda_i) - sinh(\lambda_i)}\left[sin(\lambda_i \xi) - sinh(\lambda_i \xi)\right] \quad , i = 1,2,3 \ldots (5)$$

$$Note\ that\ \ \lambda_1 = \frac{5\pi}{4} \quad , \quad \lambda_2 = \frac{9\pi}{4} \quad , \quad \lambda_3 = \frac{13\pi}{4}$$

Eq. (1) is converted into matrix type, assuming

$$\emptyset=\begin{Bmatrix}\emptyset_1\\ \emptyset_2\\ \emptyset_3\end{Bmatrix}\quad,\qquad \boldsymbol{Q}=\begin{Bmatrix}\boldsymbol{q_1}\\ \boldsymbol{q_2}\\ \boldsymbol{q_3}\end{Bmatrix}\quad ,\boldsymbol{then}$$

$$\boldsymbol{\eta}(\boldsymbol{\xi},\boldsymbol{\tau})=\emptyset^T Q=Q^T\emptyset \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(6)$$

By compensation of Eq. (6) into Eq. (1), and assuming $H=\boldsymbol{u_0^2}+\boldsymbol{\Pi}$, then

$$\emptyset^T\ddot{Q}+2M_r u_o\emptyset'^{\,T}Q^{\cdot}+H\ddot{\emptyset}^T Q+\emptyset^{(4)T}Q=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(7)$$

By multiplying $\emptyset=\begin{Bmatrix}\emptyset_1\\ \emptyset_2\\ \emptyset_3\end{Bmatrix}$ with two sides of (7) and then

$$\emptyset\emptyset^T\ddot{Q}+2M_r u_o\emptyset'^{\,T}Q^{\cdot}+H\emptyset\ddot{\emptyset}^T Q+\emptyset\emptyset^{(4)T}Q=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(8)$$

The procedure $\boldsymbol{\xi}$ integral to equation (8) within interval [0, 1], then the representation according to orthogonally for the function of trigonometric.

$$\int_0^1\emptyset\emptyset^T d\boldsymbol{\xi}=1=\begin{pmatrix}\int_0^1\emptyset_1\emptyset_1^T & \int_0^1\emptyset_2\emptyset_1^T & \int_0^1\emptyset_3\emptyset_1^T\\ \int_0^1\emptyset_1\emptyset_2^T & \int_0^1\emptyset_2\emptyset_2^T & \int_0^1\emptyset_3\emptyset_2^T\\ \int_0^1\emptyset_1\emptyset_3^T & \int_0^1\emptyset_2\emptyset_3^T & \int_0^1\emptyset_3\emptyset_3^T\end{pmatrix}d\boldsymbol{\xi}=\begin{pmatrix}\boldsymbol{1} & & \\ & \boldsymbol{1} & \\ & & \boldsymbol{1}\end{pmatrix}$$

$$\int_0^1\emptyset\emptyset'^{\,T}d\boldsymbol{\xi}=B=\begin{pmatrix}\int_0^1\emptyset_1\emptyset'^{T}_1 & \int_0^1\emptyset_2\emptyset'^{T}_1 & \int_0^1\emptyset_3\emptyset'^{T}_1\\ \int_0^1\emptyset_1\emptyset'^{T}_2 & \int_0^1\emptyset_2\emptyset'^{T}_2 & \int_0^1\emptyset_3\emptyset'^{T}_2\\ \int_0^1\emptyset_1\emptyset'^{T}_3 & \int_0^1\emptyset_2\emptyset'^{T}_3 & \int_0^1\emptyset_3\emptyset'^{T}_3\end{pmatrix}d\boldsymbol{\xi}=\begin{pmatrix}\boldsymbol{b_{11}} & \boldsymbol{b_{12}} & \boldsymbol{b_{13}}\\ \boldsymbol{b_{21}} & \boldsymbol{b_{22}} & \boldsymbol{b_{23}}\\ \boldsymbol{b_{31}} & \boldsymbol{b_{32}} & \boldsymbol{b_{33}}\end{pmatrix}$$

$$\int_0^1\emptyset\ddot{\emptyset}^{\,T}d\boldsymbol{\xi}=B=\begin{pmatrix}\int_0^1\emptyset_1\emptyset''^{T}_1 & \int_0^1\emptyset_2\emptyset''^{T}_1 & \int_0^1\emptyset_3\emptyset''^{T}_1\\ \int_0^1\emptyset_1\emptyset''^{T}_2 & \int_0^1\emptyset_2\emptyset''^{T}_2 & \int_0^1\emptyset_3\emptyset''^{T}_2\\ \int_0^1\emptyset_1\emptyset''^{T}_3 & \int_0^1\emptyset_2\emptyset''^{T}_3 & \int_0^1\emptyset_3\emptyset''^{T}_3\end{pmatrix}d\boldsymbol{\xi}=\begin{pmatrix}\boldsymbol{c_{11}} & \boldsymbol{c_{12}} & \boldsymbol{c_{13}}\\ \boldsymbol{c_{21}} & \boldsymbol{c_{22}} & \boldsymbol{c_{23}}\\ \boldsymbol{c_{31}} & \boldsymbol{c_{32}} & \boldsymbol{c_{33}}\end{pmatrix}$$

$$\int_0^1\emptyset\emptyset^{(4)T}d\boldsymbol{\xi}=A=\begin{pmatrix}\int_0^1\emptyset_1\emptyset_1^{(4)T} & \int_0^1\emptyset_2\emptyset_1^{(4)T} & \int_0^1\emptyset_3\emptyset_1^{(4)T}\\ \int_0^1\emptyset_1\emptyset_2^{(4)T} & \int_0^1\emptyset_2\emptyset_2^{(4)T} & \int_0^1\emptyset_3\emptyset_2^{(4)T}\\ \int_0^1\emptyset_1\emptyset_3^{(4)T} & \int_0^1\emptyset_2\emptyset_3^{(4)T} & \int_0^1\emptyset_3\emptyset_3^{(4)T}\end{pmatrix}d\boldsymbol{\xi}=\begin{pmatrix}\boldsymbol{\lambda_1^4} & & \\ & \boldsymbol{\lambda_2^4} & \\ & & \boldsymbol{\lambda_3^4}\end{pmatrix}, \ldots\ldots\ldots..(9)$$

The specific boundary conditions are $\boldsymbol{\emptyset_1}$, $\boldsymbol{\emptyset_2}$ and $\boldsymbol{\emptyset_3}$ which are the first three mode functions.

For pipes pinned at both ends, the B and C matrixes are:

$$B=\begin{pmatrix}0 & -2.6667 & 0\\ 2.6667 & 0 & -4.8\\ 0 & 4.8 & 0\end{pmatrix}\quad ,\ C=\begin{pmatrix}(\pi^2) & 0 & 0\\ 0 & -(2\pi^2) & 0\\ 0 & 0 & -(3\pi^2)\end{pmatrix}$$

For fixed pipes at both ends, the matrix B and C are:

$$B=\begin{pmatrix}0 & -3.3421 & 0\\ 3.3421 & 0 & 5.5161\\ 0 & 5.5161 & 0\end{pmatrix}\quad ,\ C=\begin{pmatrix}-12.3028 & 0 & 9.7315\\ 0 & -46.0501 & 0\\ 9.7315 & 0 & -98.9047\end{pmatrix}$$

For pipes pinned at one end and fixed at another end, the matrix B and C are:

$$B=\begin{pmatrix}0 & -2.9965 & 0.3167\\ 2.9965 & 0 & -5.1568\\ -0.3167 & 5.1468 & 0\end{pmatrix}$$

$$C=\begin{pmatrix} -11.5126 & 4.2814 & 3.7993 \\ 4.2814 & -42.8964 & 7.81913 \\ 3.7993 & 7.8191 & -94.0376 \end{pmatrix}$$

Using (9), after the reduced order through (8), the discretized equation is shown below:

$I\ddot{Q} + 2M_r u_0 B\dot{Q} + (CH + A)Q = 0$ ………………………………(10)

Where $\ddot{Q} = \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{Bmatrix}$, $\dot{Q} = \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{Bmatrix}$ $Q = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix}$

When $\dot{Q} = \Omega i$, $\ddot{Q} = -\Omega^2$ and Eq. (10) become;

$[-I\Omega^2 + 2M_r u_0 B\Omega i + (CH + A)]Q = 0$ ………………………………… (11)

$[-I\Omega^2 + 2M_r u_0 B\Omega i + (CH + A)]Q = S = \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{pmatrix}$ ……………… (12)

Where: $s_{11} = \boldsymbol{\lambda}_1^4 + Hc_{11} + 2M_r u_0 b_{11}\Omega i - \Omega^2$

$s_{12} = Hc_{12} + 2M_r u_0 b_{12}\Omega i$

$s_{13} = Hc_{13} + 2M_r u_0 b_{13}\Omega i$

$s_{21} = Hc_{21} + 2M_r u_0 b_{21}\Omega i$

$s_{22} = \boldsymbol{\lambda}_1^4 + Hc_{22} + 2M_r u_0 b_{22}\Omega i - \Omega^2$

$s_{23} = Hc_{23} + 2M_r u_0 b_{23}\Omega i$

$s_{31} = Hc_{31} + 2M_r u_0 b_{31}\Omega i$

$s_{32} = Hc_{32} + 2M_r u_0 b_{32}\Omega i$

$s_{33} = \boldsymbol{\lambda}_1^4 + Hc_{33} + 2M_r u_0 b_{33}\Omega i - \Omega^2$

The natural frequency (Ω) is evaluated by setting | S | equal to zero.

Expansion of this determent leads to the following characteristic equation;

$\Omega^6 - k_5\Omega^5 i - k_4\Omega^4 - k_3\Omega^3 i - k_2\Omega^2 - k_1\Omega i - k_0 = 0$ ………………………… (13)

The constants , k_0 k_1, k_2 , k_3 , k_4, k_5 , depend on the boundary conditions.

The characteristic equation becomes as below if there are two modes only:

$\Omega^4 - k_3\Omega^3 i - k_2\Omega^2 - k_1\Omega i - k_0 = 0$ ………………………………… (14)

Where k_0 k_1, k_2 , k_3 are the parameter constants.

The constants, k_0 k_1, k_2 , k_3 , k_4, k_5 , of eq. (13), depend on the boundary conditions as follows:

For (Pinned-Pinned)

$k_0 = -34609.9H^3 + 0.478222 * 10^7 H^2 - 0.165195 * 10^9 H + 0.119786 * 10^{10}$

$k_1 = 0$

$k_2 = +3436.55M_r^2 u_0^2 H - 233439M_r^2 u_0^2 - 4773.04H^2 + 555683H - 0.132176 * 10^8$

$k_3 = 0$

$k_4 = -138.175H + 9546.1 + 120.608M_r^2 u_0^2$

$k_5 = 0$

For (Clamped-Clamped)

$k_0 = -51673H^3 + 0.148296 * 10^8 H^2 - 0.120933 * 10^{10} H + 0.278330 * 10^{11}$

$k_1 = 0$

$k_2 = +4481.05M_r^2 u_0^2 H - 714029M_r^2 u_0^2 - 6243.22H^2 0.134854 * 10^7 H - 0.648214 * 10^8$

$k_3 = 0$

$k_4 = -157.258H + 18922 + 166.33M_r^2 u_0^2$

$k_5 = 0$

For (Clamped-Pinned)

$k_0 = -43139.2H^3 + 0.878896 * 10^7 H^2 - 0.478995 * 10^9 H + 0.645029 * 10^{10}$

$k_1 = -0.06613238TM_r u_0 + 0.005T^2 M_r u_0$

$k_2 = +4030.57M_r^2 u_0^2 H - 416514M_r^2 u_0^2 - 5516.43H^2 + 887358H - 0.303083 * 10^8$

$k_3 = 0.003TM_r u_0 - 0.001M_r^3 u_0^3$

$k_4 = -148.447H + 13601.8 + 142.275M_r^2 u_0^2$

$k_5 = 0$

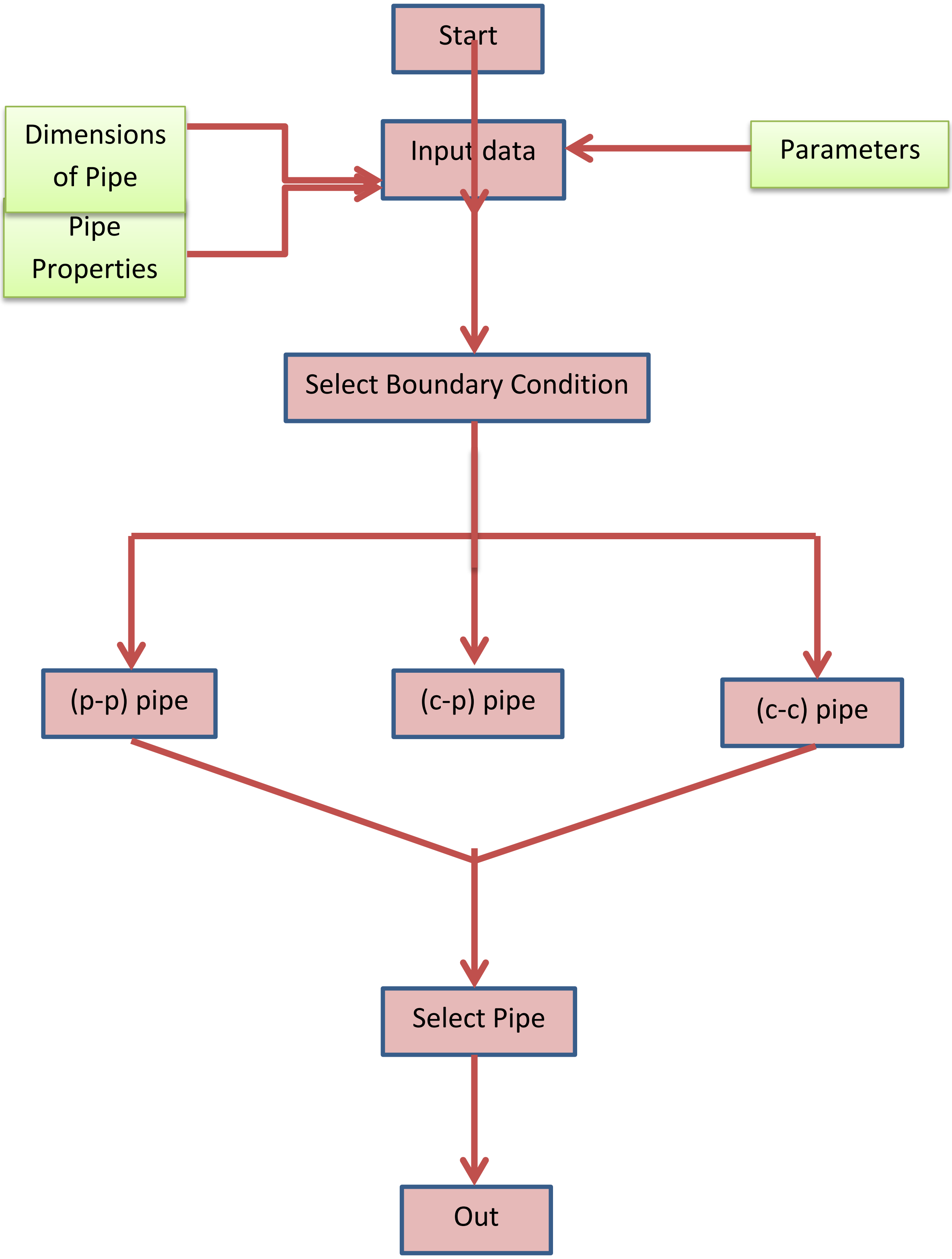

Figure. 1. The flow chart of system

3. Results and Discussions

In this section the results will discuss the theoretical results of two types of fluids (water, oil) in the various fixed cases that have been dealt with (Pinned –Pinned, Clamped- Pinned, Clamped-Clamped) and the different types of pressure (2bar, 4bar, 6bar) at constant speed. The general equation of the pipe was used where it was derived and applied to the boundary conditions for each case of fixed (Pinned –Pinned, Clamped- Pinned, Clamped-Clamped) where the program was used Matlab 2018 for programming and obtain natural frequency be without units and through the equation below.

$$\Omega = \omega L^2 \sqrt{(m_f + m_p)/EI}$$

The natural frequencies measured for the three cases of fixation of fluid-carrying pipes , pinned-pinned, clamped-pinned and clamped-clamped are shown in tables (4-1),(4-2) and (4-3) respectively .

Table 1. Pinned–Pinned pipe

Fluid is Water			
Pressure (pa)	2	4	6
Velocity 1m/s^2			
Frequency ,w (rad/sec)	119.4895	112.4565	104.9240
Velocity 2m/s^2			
Frequency ,w (rad/sec)	108.1993	100.3861	91.8822
Velocity 3m/s^2			
Frequency ,w (rad/sec)	86.4630	76.5317	65.0804
Fluid is oil			
Velocity 1m/s^2			
Frequency ,w (rad/sec)	121.2120	114.0784	106.4379
Velocity 2m/s^2			
Frequency ,w(rad/sec)	109.8308	101.9042	93.2760
Velocity 3m/s^2			
Frequency ,w(rad/sec)	87.8817	77.7968	66.1648

Table 2. Clamped-Pinned pipe

Fluid is Water			
Pressure (bar)	2	4	6
Velocity 1m/s^2			
Frequency ,w (rad/sec)	69.0095	58.0969	44.5943
Velocity 2m/s^2			
Frequency ,w (rad/sec)	51.4717	35.7640	9.4367
Velocity 3m/s^2			
Frequency ,w (rad/sec)	58.4002	45.6747	27.4765
Fluid is oil			
Velocity 1m/s^2			
Frequency ,w (rad/sec)	70.0075	58.9378	45.2403
Velocity 2m/s^2			
Frequency ,w(rad/sec)	52.2567	36.3112	9.5816
Velocity 3m/s^2			
Frequency ,w(rad/sec)	59.3978	46.4478	27.9378

Table 3. Clamped-Clamped pipe

Fluid is Water			
Pressure (pa)	2	4	6
Velocity $1m/s^2$			
Frequency ,w (rad/sec)	180.5695	175.6842	170.6474
Velocity $2m/s^2$			
Frequency ,w (rad/sec)	172.4876	167.3722	162.0840
Velocity $3m/s^2$			
Frequency ,w (rad/sec)	158.2401	152.6670	146.8723
Fluid is oil			
Velocity $1m/s^2$			
Frequency ,w (rad/sec)	183.1669	178.2121	173.1036
Velocity $2m/s^2$			
Frequency ,w(rad/sec)	175.0581	169.8696	164.5056
Velocity $3m/s^2$			
Frequency ,w(rad/sec)	160.7469	155.0925	149.2128

The tables (1,2 ,3) show the theoretical results for each type of fixations that obtained from the differential equations described previously of two types of fluids. we notice an increase in value of the frequency by increasing pressure and this indicates that the stability decreases with increasing pressure, where the lower the value.

As the internal pressure increases in the case of a pipe fixed at both ends (pinned-pinned), a decrease in the natural frequency is observed, indicating a rise in vibration amplitude. This trend is consistent across all types of boundary conditions. It has been found that, for the same pressure level, the highest natural frequency occurs in the clamped-clamped (c-c) configuration, followed by the clamped-pinned (c-p), and lastly the pinned-pinned (p-p) case. This implies that the more constrained the pipe, the higher its resistance to vibration. Therefore, the natural frequency is inversely related to the tendency of the system to vibrate.

We note from the above tables that the type of fluid affects the amount of natural frequency and the vibration of the pipe, where we notice that at the pressure of 2 bar , when the fluid is water , the frequency is 69.0095 but when the fluid is oil the frequency is 70.0075 this means that the amount of frequency increases correspondingly , the vibration in the pipe increases then the frequency continues to decreases and the vibration increases at a pressure of 6 bar . This applies to all other types of pipe installation.

Fluid velocity is important in all of the above cases, as the figures below show that the natural frequencies decrease with increasing fluid velocities, and these are real frequencies.

Given the importance of pressure in pipes used in real projects, it is also noted from the figures below that the extracted frequencies decrease with increasing pressure, reaching their lowest value at 6 bar, where the highest vibration occurs.

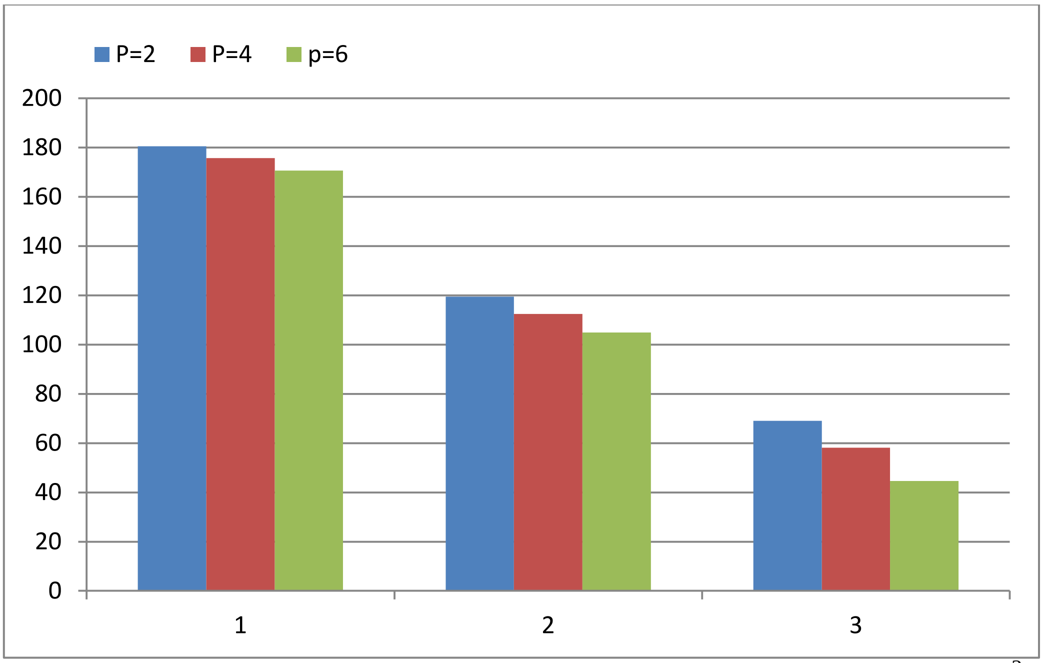

Figure 2. Comparison of different types for fixations (fluid is water at u=1m/s^2).
(1) for c-c, (2) for c-p and (3) for p-p

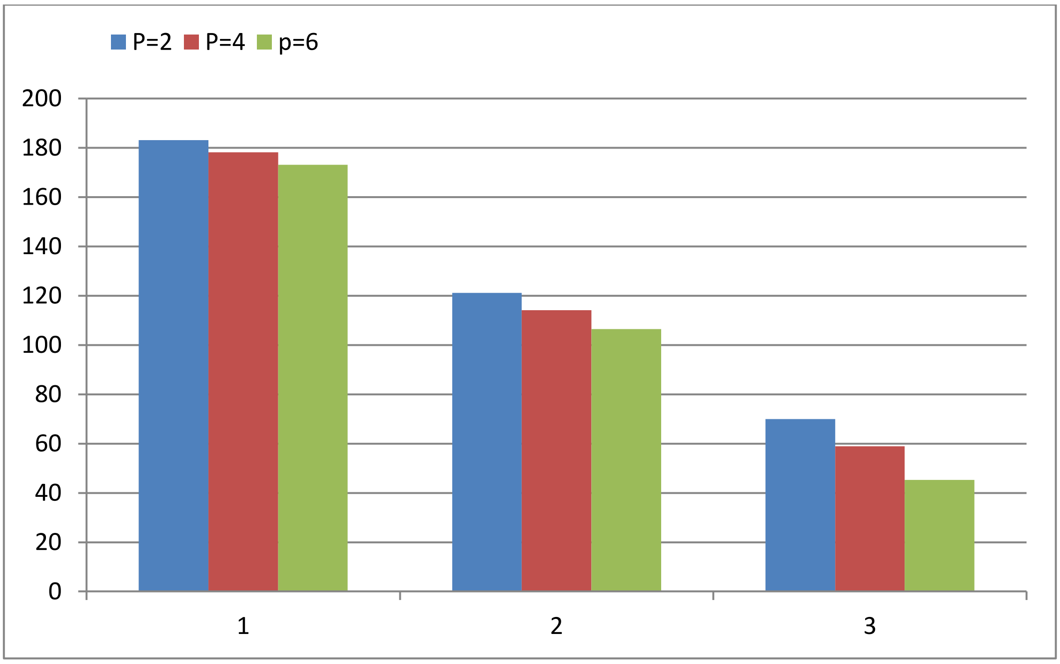

Figure 3. Comparison of different types for fixations (fluid is Oil at u=1m/s2).
(1) for c-c, (2) for c-p and (3) for p-p

Discussing the theoretical results led to an understanding of the vibration that occurs, the process of controlling it, and achieving good results with different installations.

4. Conclusions

1. The theoretical method yielded logical results for calculating frequencies at different fluid velocities and fluid specifications. This provides flexibility in extracting results. Theoretically, several installation scenarios (clamped-clamped, clamped-clamped, and clamped-clamped) were used and derived from each other. Specific boundary conditions were applied for each scenario, and different pressures (2 bar, 4 bar, 6 bar) were used at different speeds.
2. In all types of pipe installations, a decrease in natural frequencies was found as fluid flow velocity increased. In all types of pipe installations, as velocity increases, pressure increases, and vibration

increases, resulting in a decrease in natural frequencies. As velocity increases, control performance decreases due to the increased Coriolis force.

3. We note that the type of installation affects the frequency and vibration. It was found that the same amount of pressure produces the highest natural frequency for the (c-c) tube, followed by the (c-p) tube, followed by the pin-mounted tube on both sides. This means that the more the tube is restricted, the lower the vibration. The natural frequency is inversely proportional to the vibration magnitude.
4. In all types of tubes, increasing speed leads to lower natural frequencies and increasing pressure.

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